PRACTICAL ASPECTS OF POSTTENSIONING TO CONTROL END-REGION CRACKS IN PRETENSIONED GIRDERS

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ABSTRACT

Web-splitting cracking during and immediately after prestress transfer is an ongoing problem in the end region of pretensioned concrete beams. Current practices offer a variety of methods for controlling end-region cracking. This paper addresses a developing method with potential for preventing end-region cracking using vertical end-region posttensioning. Specifically, practical aspects associated with end-region posttensioning using threaded rods are discussed. These aspects include: i) application and validation of posttensioning force, ii) details for posttensioning rods, and iii) the effects of posttensioning losses. An experimental case study is also discussed, in which vertical posttensioning was successfully applied to inverted-tee beams. Variables in the case study included the quantity of posttensioning rods and the level of posttensioning. Results indicate that end-region posttensioning can reduce and delay web-splitting cracking in inverted-tee beams.

Keywords: Web-splitting, End-region, Cracking, Post-tensioning, Construction, Research

INTRODUCTION

Web-splitting cracks (Figure 1) are horizontal end-region cracks that form in the web of the prestressed girder immediately or shortly after prestress transfer. This paper focuses on a developing treatment method that may control and prevent the formation of web-splitting cracks using vertical end-region posttensioning. These cracks form due to vertical tensile stresses caused by the distribution of prestressing forces from the bottom flange to the concrete cross section. Currently, some degree of web-splitting cracks are expected and accepted in pretensioned girders. To control end-region cracks, AASHTO LRFD[1] requires placement of vertical mild steel reinforcement in the end region designed to resist a force of at least 4% of the prestressing force. Stress in the reinforcement is limited to 20 ksi. Cracks can also be controlled by reducing concrete stresses using methods such as partial strand debonding and end blocks. When cracking is severe, treatments such as patching, penetrant sealers, and epoxy injections can be applied. Vertical end-region posttensioning may serve to prevent the stresses causing web-splitting cracks, and thus challenge the status quo of treating and controlling symptoms.

Figure 1. End-region web-splitting cracks enhanced in blue[2]

Web-splitting cracks have received a good deal of attention in the literature. Previous research has focused on the causes,[3,4] design models,[5–8] reject/repair criteria,[9] and control of web-splitting cracks.[5,10,11] A recently published study by Ross et al.[2] included a proof-of-concept specimen using end-region posttensioning that had reduced cracking relative to specimens using other web-splitting control methods. Research presented in this paper adds to the body of knowledge by presenting a force calibration method for using turn-of-the-nut tightening to induce end-region posttensioning. This method provides a means of force estimation that can be readily employed in a precast fabrication facility. This procedure was successfully implemented on inverted-tee beams, and the results provide insight into the details used in end-region posttensioning, the level of posttensioning required, and the effects of time-dependent posttensioning losses.

APPLICATION AND VALIDATION OF POSTTENSIONED FORCE

Posttensioning can be applied to provide a clamping force to the end region of a prestressed concrete beam opposing the web-splitting tension developed due to the eccentric pretensioning (Figure 2). Implementation of this concept requires a method for quantifying the level of posttensioning being applied and details for the posttensioning hardware. This portion of the document presents the development of the turn-of-the-nut method for estimating the magnitude of posttensioning forces. The development was supported through an experimental program on small (typically 5.5x8 in. cross-section) concrete test specimens. Additionally, several other small specimens were created to provide insight into design details. A more extensive discussion of these specimens and testing procedures may be found in the supporting document by Willis.[12]

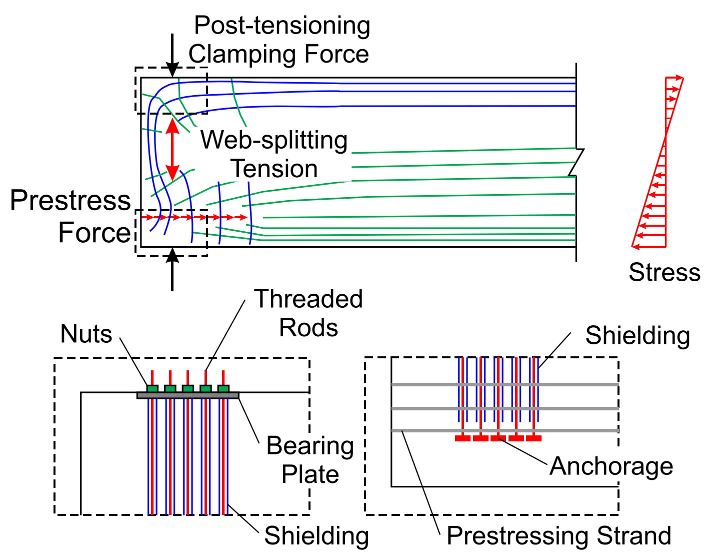


Figure 2. Web-splitting stresses and turn-of-the-nut posttensioning concept

TURN-OF-THE-NUT METHOD

While posttensioning forces can be applied to the end region of the beam through a variety of methods, the turn-of-the-nut method utilizes readily available equipment and easily applied techniques. In this method, unbonded threaded rods are anchored in concrete at one end and tensioned by tightening a nut against a bearing plate at the opposite end. The displacement of the threaded rod relative to the bearing plate can be estimated by dividing the number of turns by the rod thread count. This displacement can be used in a stress-strain model to calculate the force applied to the system.

Initial seating of the system—the tightening of nuts, plates, and washers until they are fully engaged—is a detail-specific process and complicates the behavior of the system. Because of variations in seating due to details and materials, it is not always obvious how much tightening of the nut is required to overcome seating losses. Thread conditions and seating elements in the assembly can vary between members and affect the accuracy and repeatability of posttensioning application. In investigating these effects, the test program included the use of a torque wrench to determine a consistent starting point for counting the turns of the nut and fully engaging the system beyond seating losses.

EXPERIMENTAL EVALUATION

Specimens with embedded PVC pipes (P series) were used to evaluate different procedures for applying and measuring turn-of-the-nut tensioning (Figure 3). Development of a consistent procedure was necessary before many of the other variables could be evaluated. Procedural elements under consideration included how the torque wrench was applied and the amount of torque used. Rod diameters of 1/4 in., 3/8 in., and 1/2 in. were used to evaluate the effect of diameter on tensioning procedures. Rods were made of Type 316 stainless steel with a yield strength of 84 ksi. Beam lengths were chosen to simulate different shielding lengths in a concrete section. A load cell was used to record the force in each rod during the posttensioning process. Force data from the load cell were used as a baseline to evaluate different procedures. The load cell was calibrated both before and after testing to ensure data accuracy. The experience developed in testing the P series was employed to create a calibration and application procedure. Once developed, the procedure was tested by individuals untrained in the posttensioning procedure to ensure that the process was easily replicable.

Figure 3. Test frame and data acquisition (left) and bearing details for the live end (top right) and anchor end (bottom right)

POSTTENSIONING MODEL

It was found that tensile elongation in the rod was insufficient to describe the behavior of posttensioning application. Small deformation of the anchorages at both ends must also be considered. Equation 1 was developed for estimating the posttensioning force, while including the stiffness of the rods and anchorages as a system.

$$F = turns \cdot K_s + F_{tw}$$

Equation 1

where:

F = force desired (lb.)
$turns$ = number of turns of the nut
K_s = system stiffness (lb./turn)
F_{tw} = tensile force in the rod due to seating with the torque wrench (lb.)

Load cell data was used to estimate both the initial tension placed into the system using the torque wrench (F_{tw}) and the load per turn (K_s). Figure 4 provides a graphical representation of the model. F_{tw} was determined from the load cell after the torque was applied. K_s was found using a linear regression of the load vs. turns data. Table 1 details several tests run on the P-series specimens to develop a statistical representation of the number of turns required to achieve approximately 50% of the yield strength for several distinct testing configurations.

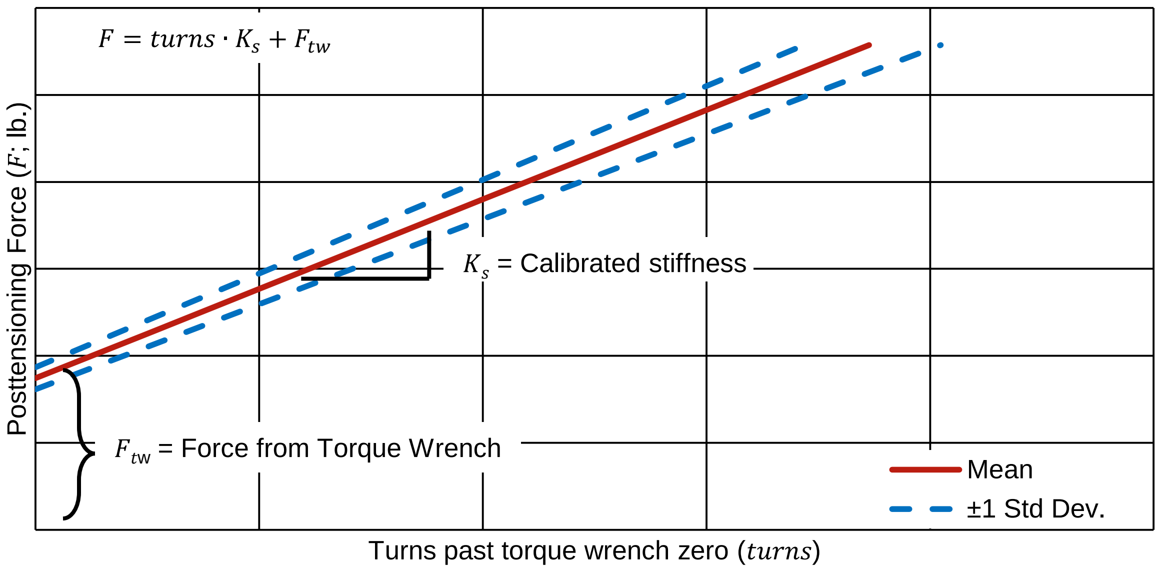


Figure 4. Calibrated stiffness model using turn-of-the-nut

Table 1. Linear regressions values and statistical characteristics determined for three specimens

Rod diameter	3/8 in.			3/8 in.			1/2 in.		
Rod length	32 in.			40 in.			40 in.		
Tests	4			8			6		
Linear Reg. (Equation 1)	K_s	F_{tw}	r^2	K_s	F_{tw}	r^2	K_s	F_{tw}	r^2
Average	2569	871	0.991	2249	1022	0.990	5266	1236	0.994
Standard Deviation	125	64	0.005	172	136	0.009	133	267	0.004
COV	0.049	0.073	0.005	0.077	0.133	0.009	0.025	0.216	0.004

APPLICATION METHODOLOGY DISCUSSION

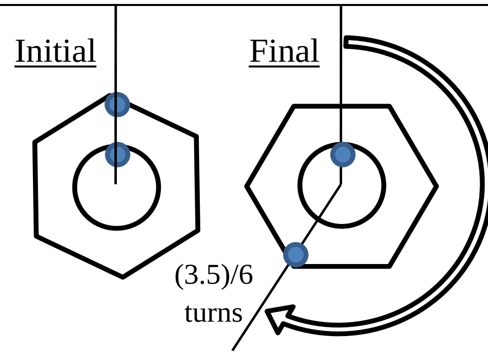
The experimentally determined parameters for the system stiffness were found to vary when system details such as anchorages, bar length, and bar diameter were also different. Therefore, the calibration process was detail specific and a separate calibration was required for each configuration. A set of instructions describing the calibration and application process is shown in Figure 5. These instructions have been tested and revised based on feedback from individuals who were initially untrained with the process. The final instructions could be implemented by an untrained third party who was familiar with basic tool usage and the operation of a load cell and torque wrench.

The intent of these directions is to assist others seeking to calibrate the turn-of-the-nut tensioning method for specific detail configurations. Once this calibration is completed, the turn-of-the-nut procedure can be quickly applied in a precast facility.

Sequential tensioning of rods affect elastic losses in the rods tensioned first. This condition was not significant in the test program because the posttension forces resulted in extremely small concrete displacement, and consequently, negligible elastic losses in the previously stressed rods. Elastic losses may be significant in other situations and should be considered in the development of a calibration procedure.

Turn count:

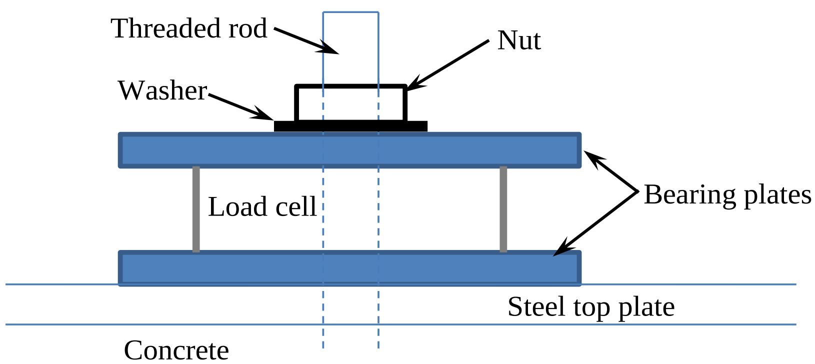
It is necessary to count the number of turns of a nut to estimate the force in the threaded rod. Counting the number of sides of the nut (6 sides in all) is a convenient method of counting partial turns. The figure to the right is an example of how turns may be counted using the faces of a nut



Calibration:

1. Set up load cell and voltmeter
2. Clear load cell bearing surface of all concrete and debris
3. Place bearing plate onto the beam if a uniform surface is not available
4. Place load cell on top of bearing plate or beam, place 2nd bearing plate on top of the load cell, and place washers and thread nut until the assembly is snug
5. Use torque wrench to apply 100 lb torque
6. Mark reference point on nut and rod threads (do not use washers as they may rotate)
7. Apply posttensioning using turn of the nut until the desired load is achieved; monitor load using load cell readout
8. Count the number of turns required to the nearest 12th of a turn (1/2 of a side on a 6 sided nut) using reference points
9. Remove nut, bearing plates, and load cell

Calibration configuration



Apply post tensioning to each rod:

1. Place washer and thread nut onto the assembly
2. Use torque wrench to apply 100 lb torque to the nut
3. Mark reference point on nut hex and rod threads (do not use washers as they may rotate)
4. Apply the number of turns determined in calibration to achieve post-tension force
5. Repeat procedure on all bolts

Figure 5. Calibrated force estimate instructions

DESIGN DETAILS

The end-region posttensioning concept requires anchorage and shielding of the threaded rods. Small specimens similar to those shown in Figure 3 were used to evaluate anchorage and shielding details for 1/4 in. and 1/2 in. diameter rods. Anchorages included hooked rods, straight rods, and rods with a nut and washer threaded on the end similar to headed bars. No significant differences were observed between the anchorage configurations, and all were able to resist the applied forces and allow for yielding the rods. Rods were shielded using plastic wrapping or duct tape. Similarly, shielding was found to have no observable effect on the performance of the rods.

In the inverted-tee beam specimens discussed later in the case study, rods used standard hooks and were shielded using plastic strand shielding common to precast plants. To comply with ACI requirements and streamline rod production, it is suggested that post-tension rods be bent using standard bar bend procedures. The commercially produced plastic shielding used to debond prestressing strand is recommended for shielding as it is readily available at most precast plants and was more time efficient in implementation than the other techniques tested.

CASE STUDY

APPROACH

Two 36 in. tall and 12 ft long inverted-tee beams were fabricated to evaluate the effectiveness of end-region posttensioning in preventing and controlling web-splitting cracks. The beams had 26 strands stressed to 33.8 kips each and released using flame cutting. Reinforcement and prestressing are shown in Figure 6 and Figure 7. Beams were fabricated a precast facility in South Carolina. They were fabricated on the same bed and at the same time as production beams.

The beam design was based on a production beam known to develop web-splitting cracks. Construction details and an extended discussion of the procedures and results can be found in the supporting document by Willis.[12] Posttensioning was applied using the calibrated turn-of-the-nut method to Type 316 stainless steel rods. The two test beams were produced with varying details at each end, resulting in four different end-region designs. In this paper each end region is referred to as a unique specimen. Specimens included two controls, CT1 and CT2. CT1 had the same web and flange reinforcement as the production beams with no posttensioning rods or anchorage plates. CT2 had the same reinforcement as CT1, but also had posttensioning rods and anchorage plate but without any posttensioning force. PT1 and PT2 were two end regions with posttensioning rods and anchorage plates but different posttensioning forces (4.3% and 5.5% respectively). With the exception of the posttensioning rods, reinforcement in PT1 and PT2 was identical to the control CT1. The test matrix can be seen in Figure 8 where the posttensioning is reported as a percent of initial prestressing force. Posttensioning details can be seen in Figure 6 and Figure 9. Anchorages consisted of hooked rods at the base of the beam and a ¼ in. thick steel bearing plate at the top. Shielding was provided by plastic shielding material used for pretensioned strands.

Posttension rods were placed as close to the end as practical. This placement was effective in the test specimens but similar placement affected diagonal web cracking in the I-girder specimens by Ross et al.[2]. Principal stresses in the concrete should be considered when designing the magnitude and placement of posttensioning.

Figure 6. End-region details: (left) mild steel reinforcement for the control specimen, (top right) bearing plate and threaded rods with strand shielding, and (bottom right) hooked anchorages for threaded rods

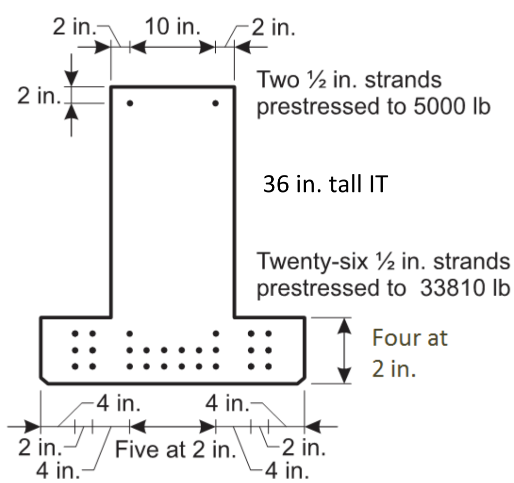


Figure 7. Strand layout and prestressing detail

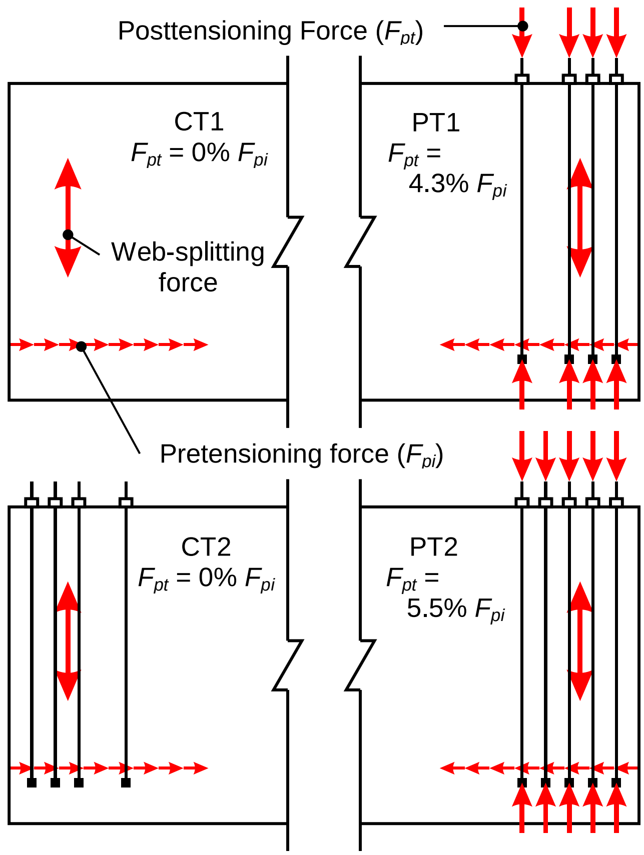


Figure 8. Variable matrix with posttensioning forces in percent of initial prestressing force

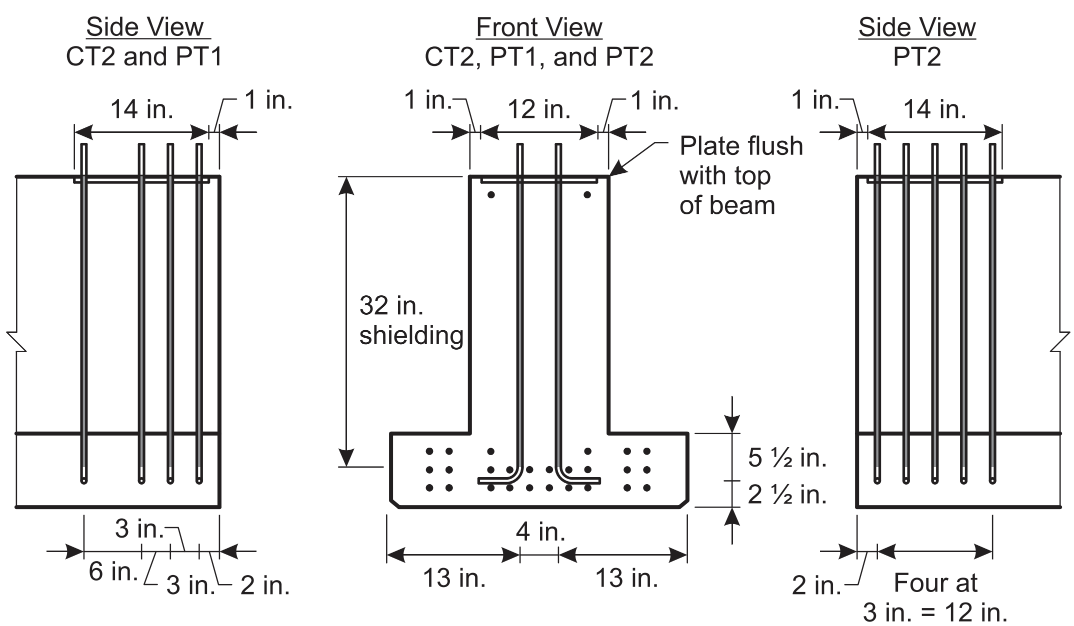


Figure 9. Posttensioning details for specimens CT2, PT1 and PT2

As discussed in the previous section, calibrated turn-of-the-nut method consists of using a torque wrench and a load cell to determine the number of turns required to achieve the desired posttensioning load. A calibration was conducted on the test specimens, and then the load cell was removed and all of the rods in PT1 and PT2 were tightened using the torque wrench and the determined number of turns. The beams were monitored for cracking during the days and months following fabrication.

Visual inspections were used to collect crack data to evaluate effectiveness of the posttensioning schemes. Data included crack lengths and widths and was monitored closely for the first 28 days and then periodically for nearly nine months afterwards. Effectiveness of posttensioning was established through direct comparison of crack data from the post-tensioned specimens to control specimens.

POSTTENSIONING APPLICATION

Based on discussion with the employees at the fabricator, installation of the posttensioning rods and bearing plate was not more difficult than that of other commonly used steel reinforcement and plates. Posttensioning was applied to the threaded rods when the concrete compressive strength exceeded the specified release strength and after the forms were removed, but before the pretension force was transferred. The calibrated estimates for the posttensioning force applied to specimens PT1 and PT2 were equal to 4.3% and 5.5% of the measured (not specified) pretensioning force or 38.9 and 52.3 kips respectively. These levels of posttensioning differed from the target 4% and 6% because of the actual prestressing stress levels. The target values were chosen based on AASHTO requirements[1] that crack control in the end region be designed for 4% of the pretension force. PCI recommendations[13] were for 3% of the prestressing force for this cross section if transfer length is estimated at 50 times the diameter of the strand.

To execute the calibration procedure, each of the rods used for calibration had to be left with enough length to allow for the load cell assembly to be placed over the rod. However, the allowable length extending above the plate was limited to the depth of the socket used on the torque wrench. To accommodate this, calibration was first executed on four rods that were approximately three inches longer than the other rods. The calibration factors were consistent between rods. After calibration, the load cell was removed and the calibrated rods were trimmed to the same height as the other rods. The whole posttensioning process required an hour and three quarters. Excluding calibration, the final tensioning of all of 18 of the rods in both PT1 and PT2 took fewer than 20 minutes, which amounted to approximately one minute per rod. If a calibrated estimate for rod tensioning was already completed for this cross section and configuration, the implementation time would have been closer to the 20 minutes rather than 105 minutes.

EVALUATION OF CRACKING

Cracking was typical of web-splitting cracking as seen in Figure 10. Several terms used throughout this discussion are illustrated in Figure 10 for clarity. The cracking on each side appeared roughly symmetric and the crack traversed the face of the beam, traveling straight across between sides. The general crack profile for this distress is described as triangular with the base being at the beam end and crack widths along the beam web decreasing uniformly as they move away from the end of the beam. Figure 11 shows cracking for each specimen on the side of the web at 28 days after prestress transfer. Because all specimens except for PT2 exhibited a single crack traversing the beam end, crack lengths excluding the 14 in. width of the beam end are used as a metric for evaluating the end-region details. This metric is termed "crack length along the web" in this discussion (Figure 10). Crack widths at the end of the beams are the typically the maximum crack widths and are therefore used when discussing crack width.

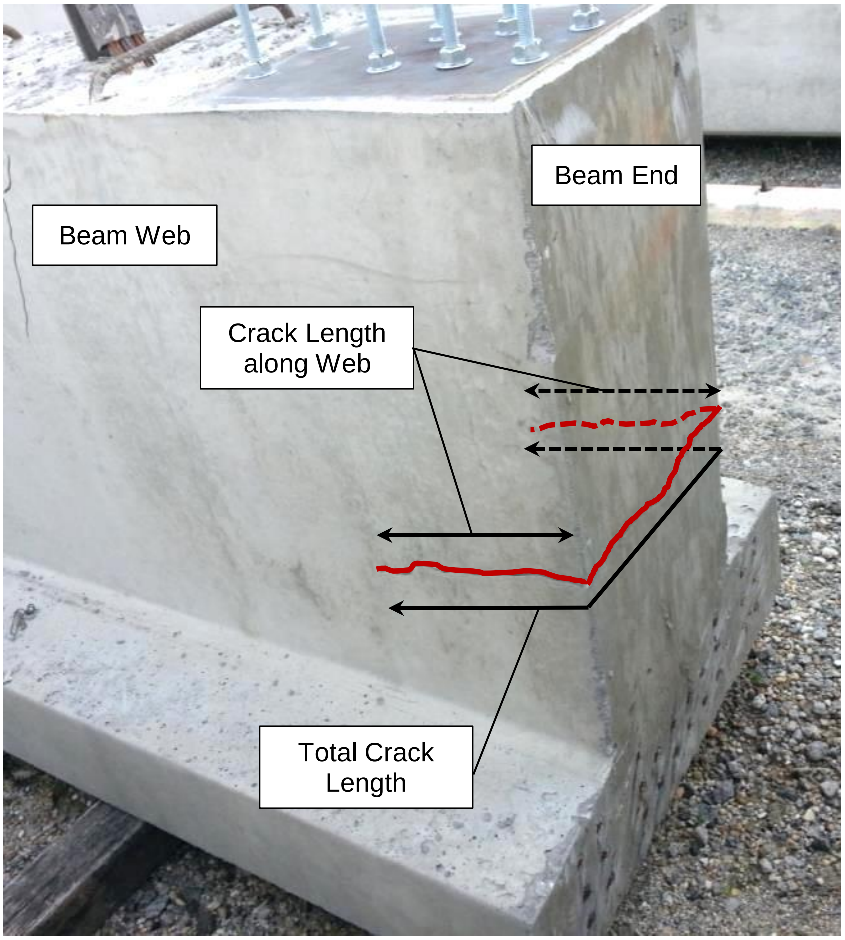


Figure 10. Typical web-splitting cracking (specimen CT2) with terms used in the discussion

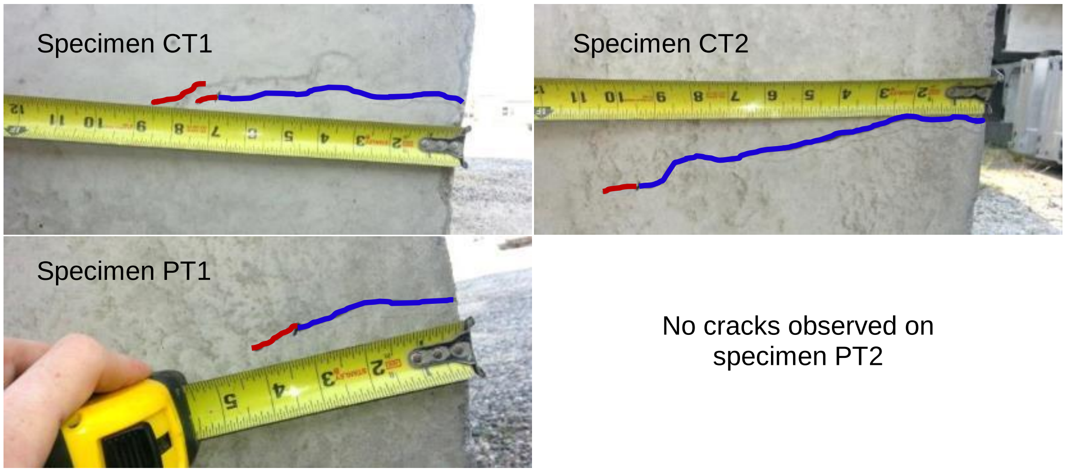


Figure 11. Cracking observed in specimens along the web during the 28 day period; blue cracks were observed immediately after lifting onto dunnage, red cracks were observed 4 days later

Specimens were evaluated for cracking immediately after prestress release, after each lifting event, and daily for the first 28 days. The beam was transported to off-site storage 76 days after prestress transfer. Observations were made periodically between the 28 day mark and 220 days after release. Crack widths and lengths for all specimens are represented in Figure 12. No cracks were observed immediately after prestress transfer, but were noted after the beams were moved to dunnage next to the prestressing line. Inspections continued after the beam was moved to outside storage and some crack lengthening was noted 4 days after transfer. No additional crack growth was noted until 180 days after prestress transfer.

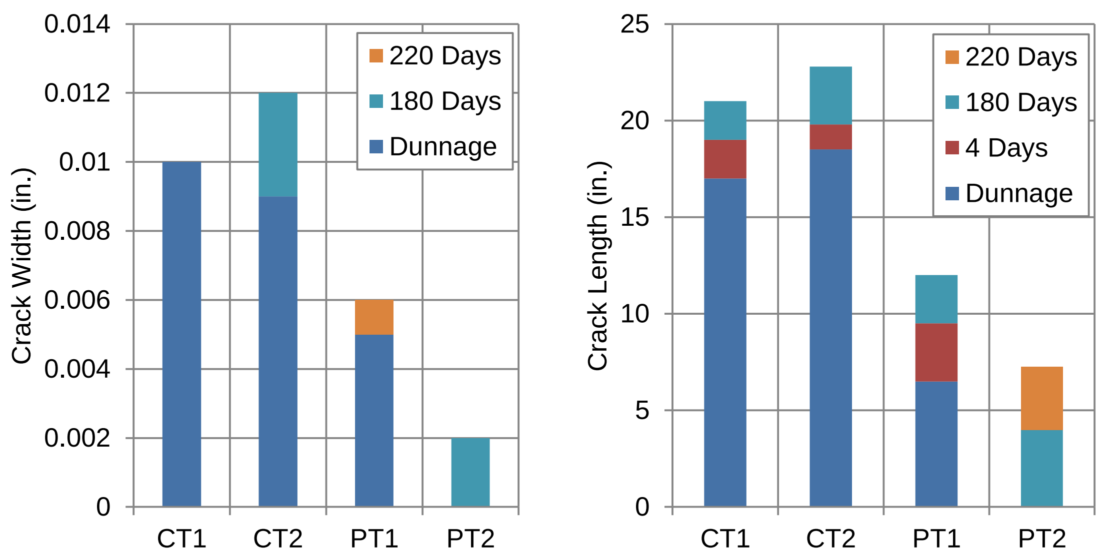


Figure 12. Specimen crack width (left) and crack length along the web (right) with date of observation

PT1 and PT2 had crack widths equal to 55% and 18% of the control specimens, respectively. The crack lengths along the web for PT1 and PT2 were also reduced to 55% and 23% of the control specimens. The 5.5% of prestressing force provided in specimen PT2 was effective at delaying cracking until 180 days after fabrication and was also more effective than specimen PT1 at controlling crack width. Crack growth in specimen PT2 after 180 days could be indicative of posttensioning losses. Continued crack growth has been observed by other researchers,[14] but was not as significant in these specimens. Overall, both posttensioned specimens controlled crack widths and reduced the total length of cracking compared to the controls. Additional tests are required to determine if greater levels of posttensioning would eliminate the cracks totally.

TIME-DEPENTDANT LOSS OF PRESTRESS

As in all prestressed concrete structures, there is need to consider immediate and time-dependent loss of prestress when designing end-region posttensioning. Immediate losses due to seating and elastic shortening can be addressed using the calibrated turn-of-the-nut methodology presented earlier. Time-dependent losses are more difficult to addresses. For the inverted-tee specimens, there was concern that time-dependent effects might cause a significant loss of prestress. Crack growth in specimen PT2 between 180 and 220 days after release validated this concern. To evaluate loss of posttension, the nuts on specimens PT1 and PT2 were loosened 220 days after fabrication. Strain data and crack data were collected to determine if the rods were still in tension after 220 days and if loosening of the nuts relieved that tension.

Strain data were collected while loosening of the nuts using vibrating wire strain gages attached to the end face of the specimens. Figure 13 shows the gage locations for specimens PT1 and PT2. As the nuts were loosened the strain in the concrete grew in tension, indicating loosening the nuts did affect a detensioning of the rods. The strain data (Figure 14) confirm that the posttensioning force was continuing to provide clamping action to the end region even after 220 days.

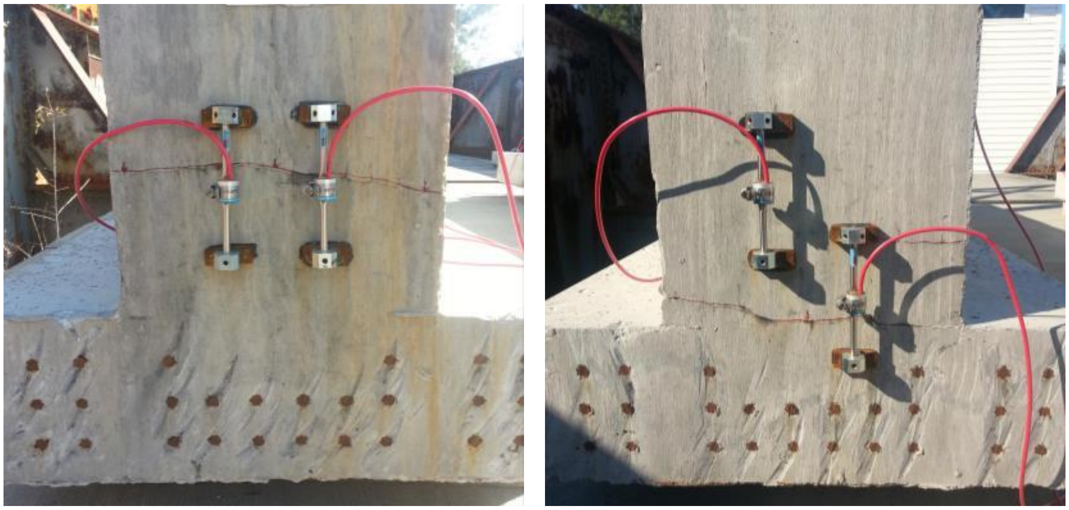


Figure 13. VWG placement for PT1 (left) and PT2 (right); specimen cracking at 220 days with cracks enhanced for visibility

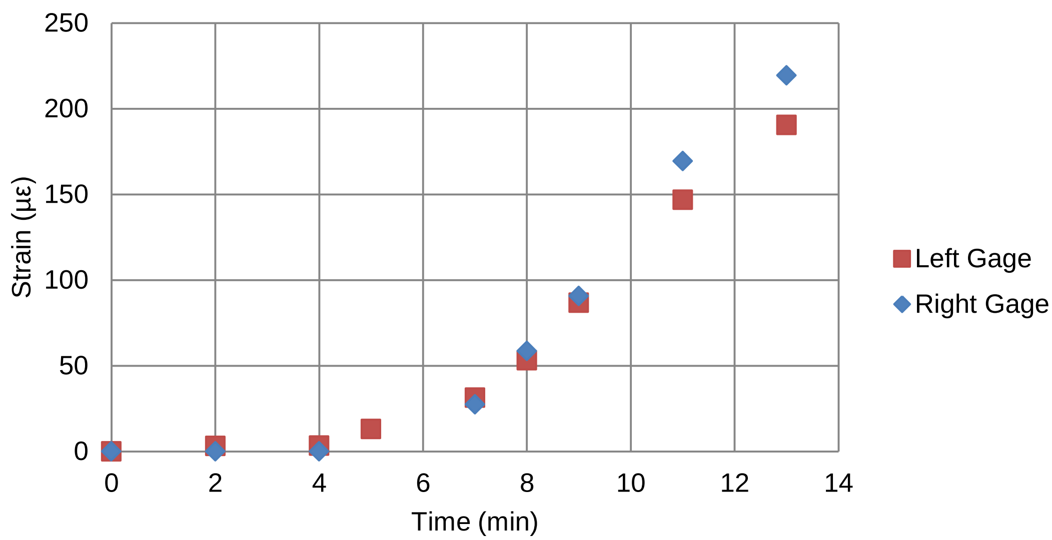


Figure 14. Vertical strain in specimen PT1 during detensioning; after an initial value, each data point was taken after a nut was removed. Similar results were obtained from PT2.

Crack growth was observed in PT1 and PT2 after the nuts were loosened. This observation confirms the previous conclusion from the strain data that the rods were in tension prior to loosening the nuts. While the VWGs indicated immediate strain changes in the system, cracks were not observed to grow until approximately 7 hours after detensioning. No additional growth was observed after the initial 60 hours of observation. The width and length of cracks in PT1 grew by 50% and 73%, respectively. For PT2, the width grew 67% and the length grew 76% (Figure 15).

Cracks in PT1 and PT2 grew in length and width, but did not reach the level of cracking observed in the control specimen. PT1 grew to 82% and 95% of the average width and length of the control specimens CT1 and CT2. PT2 grew to 58% and 38% of the controls lengths and widths. The reduction in ultimate crack lengths and widths in the PT specimens when compared to the CT specimens is attributed to the increased concrete strength at 220 days relative to the concrete strength at release. The posttensioning forces in PT1 and PT2 provided clamping at the time when the concrete was weakest and cracking was most likely. After detensioning the concrete was stronger and more able to resist cracking.

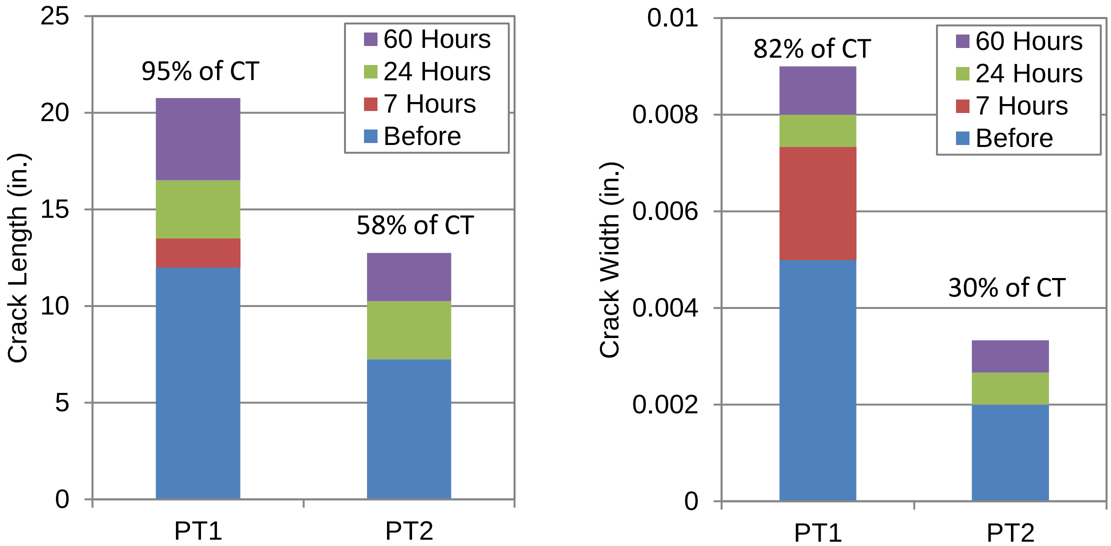


Figure 15. Crack growth during the detensioning process in both specimens PT1 and PT2; final crack lengths and widths are compared to the average results from the control specimens

Based on the strain and crack observations, it is concluded that the threaded rods maintained prestress force until they were detensioned at 220 days. Thus it can be qualitatively stated that time-dependent effects did not cause a complete loss of prestress. The available strain and crack data are considered too limited and too imprecise to make a meaningful quantitative estimate regarding the magnitude of time-dependent losses. Quantifying time-dependent losses in end-region posttensioning applications is recommended as a topic for future research.

Time-dependent losses in PT1 and PT2 are believed to have occurred primarily due to shrinkage in the concrete and relaxation of the steel threaded rods. Creep losses in the concrete are presumed to be negligible because creep is a phenomenon associated with concrete stressed in compression. The occurrence of cracks in PT1 and PT2 prior to detensioning (Figure 13) indicates that the end-region was in tension and that creep did not occur.

The threaded rods in specimens PT1 and PT2 were Type 316 stainless steel with a yield strength of 84 ksi. Because the effects of relaxation increase at stresses close to yielding, the prestress in the threaded rods was limited to 42% of yielding. The use of high-strength rods to address relaxation losses is suggested in future applications. Additional tightening of the nuts during the storage period of the member is also suggested as a means of addressing relaxation losses.

Most of the cracking in the beam specimens occurred within hours of prestress transfer (Figure 12), when the concrete strength was still relatively low. Increases in concrete strength after transfer occur simultaneously with loss of prestress in the end-region posttensioning. The balance between increasing concrete strength and prestress loss is another topic for future investigation.

SUMMARY AND RECOMMENDATIONS

APPLICATION AND VALIDATION OF POSTTENSIONING FORCE

A posttensioning turn-of-the-nut procedure was developed to provide clamping forces to the end region of a prestressed concrete beams to resist web-splitting cracking. Posttensioning procedures and details were developed and evaluated using two series of small specimens. The objective of testing was to develop a turn-of-the-nut posttensioning procedure for applying consistent posttensioning forces. Conclusions were drawn on the effect of anchorage, shielding, and rod sizes on the calibrated estimation of the posttensioning force and are summarized below.

- A calibrated turn-of-the-nut procedure can be employed to apply consistent and accurate forces by estimating the system stiffness for a specific set of details.
- Hooked threaded rods bent to ACI specifications[15] were effective in the test program. Headed bars may also be effective but should be tested for local cracking of concrete around the heads prior to use.
- Plastic shielding material used for pretensioned strands is recommended to debond threaded rods from the concrete.
- Smaller diameter rods (1/4 in.) are not recommended.

CASE STUDY

Posttensioning was applied using the calibrated turn-of-the-nut estimation method to two inverted-tee beams. The objective of testing was to evaluate the application process in a precast plant environment and to quantify the effectiveness of end-region posttensioning. Crack lengths and widths were monitored by visual inspection over the days and months following prestress release. Conclusions pertaining to the application and effectiveness are summarized below.

- Application of posttensioning force can be readily accomplished in a prestressed facility using the calibrated turn-of-the-nut method. After the calibration was established, it took an average of one minute per rod to apply posttensioning force to each specimen.

- Posttensioning can be an effective means of delaying and controlling web-splitting cracking. For the test specimens, 4.3% of the prestressing force was not sufficient to prevent immediate cracking, but did control cracks more effectively than control specimens. Applying 5.5% of the prestressing force delayed cracking until 180 days after prestress transfer.
- Posttensioning was effective in reducing crack width and length in the test specimens. Both crack width and length in PT1 was reduced relative to the control specimens by 55%. Crack width and length in specimen PT2 was reduced relative to the control specimens by 18% and 23% respectively

TIME-DEPENDENT LOSSES

Posttensioning was removed after 220 days to determine whether posttensioning forces were continuing to confine crack growth. Immediate strain changes and crack growth support the conclusion that a degree of posttensioning remained at 220 days despite time-dependent losses. Shrinkage of the concrete and relaxation of the steel are suspected to be the primary components of posttensioning losses.

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